METHODS OF CALCULATION OF DIGITAL SIGNALS SPECTRA

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Abstract. *The signal is a physical bearer of information (electric or optical energy, electromagnetic or air waves) which changes in course of time. Only a random signal the height of which in certain time instants can be anticipated with a probability 0 < p < 1 conveys information. The extreme cases perform the white noise on one hand, the value of which can not be anticipated at all (p = 0), and a constant or a periodic deterministic signal on the other hand, the values of which are known in each time instant with probability p = 1. Such signals do not carry any information. Typical information bearers in telecommunication techniques are digital signals that can be classified as random ones, discrete in time and in amplitude. As they are performed by a train of pulses with random amplitudes, they contain a periodic deterministic component. Because they are random, they can only be described by statistical characteristics as the mean value, the dispersion, the power and by the more complex characteristic – the power spectral density (the power spectrum) that can be derived using tools of theory of random processes. A simpler case is a digital signal with pulses with random amplitudes without any correlation among pulses (m-PAM codes). Its power spectrum can easily be derived [1], [2], [7], [10]. It is more difficult to derive the power spectra of the random signals performed by pulses with random amplitudes and with correlation among particular pulses. This is the topic which deals this paper with. The simple convolution coding, the MLT 3 code and the AMI-NRZ code, frequently used telecommunication branch [6], [8], [9], all with the correlation coupling between pulses, are considered as an example of calculation of the power spectrum of a digital signal. Further information about codes and spectral analysis can be found in [3], [4], [5].*

Keywords

Digital signal, spectrum, crosstalk, codes, correlation.

1. Introduction

A digital signal carrying information is created by the train of periodically repeating pulses of a certain shape the amplitude of which in a k-th repeating period T_o is a random variable A_k acquiring discrete values a_{kj} with probabilities p_j, $j = 0, \pm 1, \pm 2, \ldots, \pm M/2$ where M denotes the count of discrete states of the non-zero amplitudes of a digital signal. It can be described in the time domain as:

$$X(t) = \sum_{k=-\infty}^{\infty} A_k \cdot f t) \ldots\ldots\ldots -\frac{\tau}{2} + kT_o \le t \le kT_o + \frac{\tau}{2}, \quad (1)$$

where $f(t)$ is the function with the amplitude which equals 1 and which shapes the pulses of a digital signal and τ is the width of the pulse (Fig. 1).

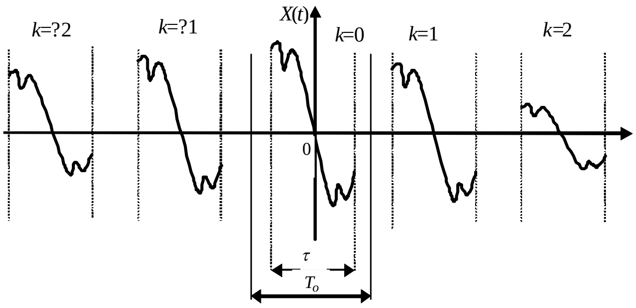

Fig. 1: An example of a digital signal.

The complex power spectral density $\boldsymbol{S}(f)$ of a random signal occurring as a train of periodically repeating pulses of a certain shape is according to [7] given by:

$$\boldsymbol{S}(f) = \frac{|\boldsymbol{F}(f)|^2}{T} \sum_{n=-\infty}^{\infty} R_n \mathrm{e}^{-\mathrm{j}2\pi f T_o} + m_a^2 \sum_{v=-\infty}^{\infty} |\boldsymbol{c}_v|^2 \delta(v \cdot f_o) \,. \quad (2)$$

In the equation (2),

$$\boldsymbol{F}(f) = \int_{-\frac{\tau}{2}}^{\frac{\tau}{2}} f(t) \cdot \mathrm{e}^{-\mathrm{j}2\pi f t} \mathrm{d}t \,, \quad (3)$$

is the Fourier transform of the function $f(t)$,

$$c_v = \frac{1}{T}\int_{-\frac{\tau}{2}}^{\frac{\tau}{2}} f(t)\cdot e^{-j2\pi v f_o t}\,dt\,, \tag{4}$$

is the complex coefficient of the Fourier series of the non-random component of the random signal with pulses which are shaped according to the function $f(t)$,

$$m_a = \overline{A} = \sum_{j=-\frac{M}{2}}^{\frac{M}{2}} a_j \cdot p_j\,, \tag{5}$$

is the mean value of the random amplitude A of a pulse and $\delta(v.f_o)$ is the Dirac pulse.

In general, the correlation between pulses in a k-th and a $(k+n)$-th repeating period can be expressed as:

$$R_n = \overline{A_k \cdot A_{k+n}} = \sum_{k=0}^{N-n-1}\sum_{i=-\frac{M}{2}}^{\frac{M}{2}}\sum_{j=-\frac{M}{2}}^{\frac{M}{2}} a_{ki}\cdot a_{(k+n),j}\cdot p_{kij}\,, \tag{6}$$

for $n = 0, 1, 2, ..., N-1$ where p_{kij} is the probability of occurrence of a j-th amplitude in a $(k+n)$-th pulse on condition that an i-th amplitude has occurred in a k-th pulse:

$$p_{kij} = p_{ki}\cdot p_{(k+n),j/i}\,. \tag{7}$$

The equation between periods T and T_o is:

$$T = N\cdot T_0\,. \tag{8}$$

N denotes the correlation range, i.e. the number of pulses that may have a correlation coupling a among each other within a period T.

If there is no correlation between pulses of a digital signal but the signal has also a non-random component, formula (2) gets simpler:

$$\mathbf{S}(f) = \frac{\sigma_a^2\left|\mathbf{F}(f)\right|^2}{T} + m_a^2\sum_{v=-\infty}^{\infty}\left|\mathbf{c_v}\right|^2\delta(v\cdot f_o)\,, \tag{9}$$

where σ_a^2 is the dispersion of the random amplitude of a digital signal:

$$\sigma_a^2 = \sum_{j=-\frac{M}{2}}^{\frac{M}{2}} a_j^2\cdot p_j - m_a^2 = \overline{A^2} - m_a^2\,. \tag{10}$$

In the equations (5) and (10), p_j is the probability of occurrence of a pulse amplitude a_j.

If there is only a zero discrete coefficient, c_0 in the spectrum representing a constant component in the random signal, then

$$\mathbf{S}(f) = \frac{\sigma_a^2\left|\mathbf{F}(f)\right|^2}{T} + m_a^2 c_0^2\,. \tag{11}$$

If a signal contains only a pure alternate component, then the equation (2) can be further simplified to:

$$\mathbf{S}(f) = \frac{\sigma_a^2\left|\mathbf{F}(f)\right|^2}{T}\,. \tag{12}$$

Let´s consider the simple convolution coding, the MLT 3 code and the AMI-NRZ code all with the correlation coupling between pulses as an example of the calculation of the power spectrum of a digital signal.

2. Convolution Code

The main purpose of convolution codes is to detect and correct errors in the transmission of digital signals. Each coding which enables this brings redundancy and non-random elements into the original random digital signal in that manner that the state of some symbols depends on the state previous symbols. Thanks this redundancy, there are more symbols possible for a given input information stream. The chose of the right symbol among more possible pulses that can be taken into consideration is determined on an algorithm based on the trellis diagram which is known for both the transmitter and the receiver. The trellis diagram in Fig. 3 catches all possible states which the encoder on Fig. 2 can have, and the signal on the output of the encoder that reflects these states and that is enlarged by redundancy bits.

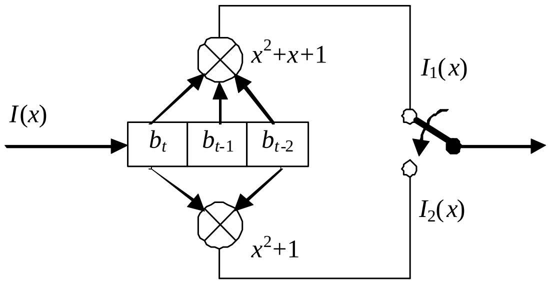

Fig. 2: Example of a convolution encoder.

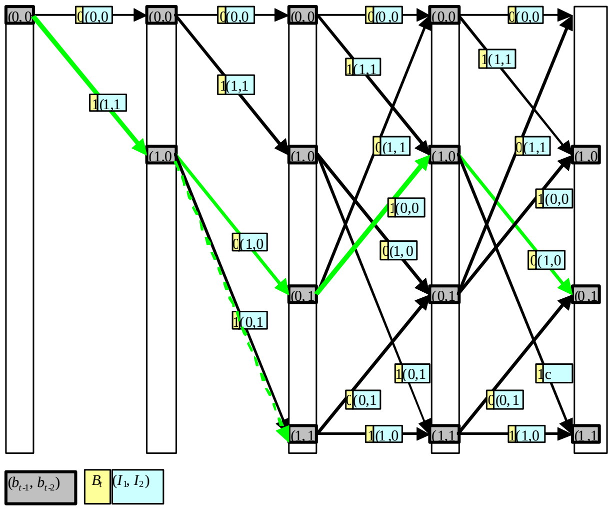

Fig. 3: Trellis diagram of the encoder on Dig. 2.

It can be found out by the analysis of transitions and states in Dig. 3 that the actual 2-bit is bound with the previous 2-bit and so they create together 4-bits from which 8 from all 16 possible combinations of 4-bits are only allowed according to Tab. 1. As it can be seen from this table, all combinations of 00, 01, 10, 11 can occur on the 2nd and 3rd place in the 4-bit. But if a 0 or a 1 occur on the 1st place, these 0 or 1 are also on the 4th place. That means that the series of 4-bits can be decomposed into 3 independent bit series according to Fig. 4:

a) a random bit series that includes bit on the 2nd place in the 4-bit,

b) a random bit series that includes bit on the 3rd place in the 4-bit,

c) a random 2-bit series that includes bits on the 1st and 4th place in the 4-bit which, creating the signal c), are correlated.

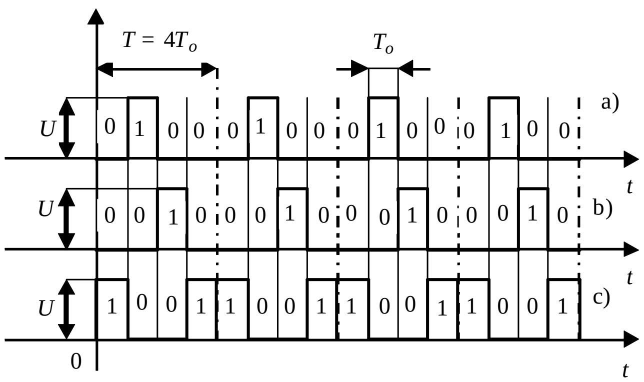

Fig. 4: Decomposition of a bit series after convolution encoding.

Tab.1: Allowed states on the output of the encoder as on the Fig. 2.

Number	Bit combinations	
	possible	allowed
0	0000	00 00
1	0001	
2	0010	00 10
3	0011	
4	0100	01 00
5	0101	
6	0110	01 10
7	0111	
8	1000	
9	1001	10 01
10	1010	
11	1011	10 11
12	1100	
13	1101	11 01
14	1110	
15	1111	11 11

The amplitude of a digital signal is a random variable A_k which acquires 2 values: $a_{k0} = 0$ with the probability $p_{k0} = 1/2$ and $a_{k1} = U$ with the probability $p_{k1} = 1/2$. Then the mean value of the amplitude will be:

$$m_a = \overline{A_k} = \sum_{j=0}^{1} a_{kj} \cdot p_{kj} = 0 \cdot \frac{1}{2} + U \cdot \frac{1}{2} = \frac{U}{2}, \qquad (13)$$

and the dispersion:

$$\sigma_a^2 = \sum_{j=0}^{1} a_{kj}^2 \cdot p_{kj} - m_a^2 = 0^2 \cdot \frac{1}{2} + U^2 \cdot \frac{1}{2} - \left(\frac{U}{2}\right)^2 = \frac{U^2}{4}. \qquad (14)$$

Let´s consider rectangular shape of the random pulse filling the whole period ($\tau = T_o$), Fig. 5. Then:

$$f(t) = 1 \ldots\ldots\ldots -\frac{\tau}{2} \leq t \leq +\frac{\tau}{2}. \qquad (15)$$

The Fourier transform (3) of that pulse is:

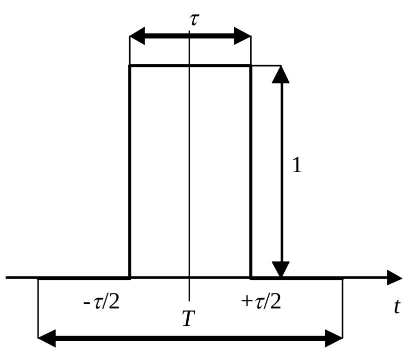

Fig. 5: Rectangular pulse.

$$\boldsymbol{F}(f) = \int_{-\frac{T_o}{2}}^{\frac{T_o}{2}} 1\cdot \mathrm{e}^{-\mathrm{j}2\pi f t}\,\mathrm{d}t = T_o \cdot \frac{\sin\frac{\pi f}{f_o}}{\frac{\pi f}{f_o}}, \qquad (16)$$

and Fourier coefficients:

$$\mathbf{c}_v = \frac{1}{T}\int_{-\frac{T_o}{2}}^{\frac{T_o}{2}} 1\cdot \mathrm{e}^{-\mathrm{j}2\pi v f_o t}\,\mathrm{d}t = \frac{T_o}{T}\cdot\frac{\sin(v\pi)}{v\pi} = \begin{cases} \frac{1}{N} \ldots v = 0 \\ 0 \quad \ldots v \neq 0 \end{cases}. \qquad (17)$$

The signal has only the constant component c_0.

1) Alternate Component of the Signal a)

The equation (16) performs the Fourier transform of the pulse. And thus the double side power spectral density of the alternate component of the signal will be given by the equation (12):

$$\mathbf{S}_1(f) = \frac{U^2}{4}\cdot\frac{1}{4T_o}\cdot\left| T_o \cdot \frac{\sin\frac{\pi f}{f_o}}{\frac{\pi f}{f_o}} \right|^2 = \left(\frac{U}{4}\right)^2 T_o \left(\frac{\sin\frac{\pi f}{f_o}}{\frac{\pi f}{f_o}} \right)^2 . \qquad (18)$$

2) Alternate Component of the Signal b)

This component is the same as the component a) and

$$\boldsymbol{S}_2(f) = \boldsymbol{S}_1(f). \qquad (19)$$

3) Alternate Component of the Signal c)

There is the correlation coupling between the 1st and 4th in the period T. Therefore the correlation coefficients R_n, $n = 0, 1, 2, 3$ will be calculated according to the equation (6) where $N = 4$ is the number of pulses in the period T (the correlation range).

For $n = 0$:

$$R_0 = \overline{A_k \cdot A_k} = \overline{A_k^2} = \sigma_a^2 + m_a^2 = \frac{U^2}{4} + \left(\frac{U}{2}\right)^2 = \frac{U^2}{2}, \qquad (20)$$

and so the complicated calculation according to (6) can be avoided for $n = 0$.

For $n = 1$, the upper boundary of the first sum $N - n - 1 = 4 - 1 - 1 = 2$ and

$$R_1 = \overline{A_k A_{k+1}} = \sum_{k=0}^{2}\sum_{i=0}^{1}\sum_{j=0}^{1} a_{ki}\cdot a_{(k+1),j} = 0, \qquad (21)$$

as there is no correlation between adjacent pulses within period T.

Similarly, for $n = 2$, the upper boundary of the first sum $N - n - 1 = 4 - 2 - 1 = 1$ and,

$$R_2 = \overline{A_k A_{k+2}} = \sum_{k=0}^{1}\sum_{i=0}^{1}\sum_{j=0}^{1} a_{ki}\cdot a_{(k+1),j} = 0, \qquad (22)$$

as there is not correlation of the 1st pulse with the 3rd one, and of the 2nd pulse with the 4th one in the period T.

Finally, for $n = 3$, the upper boundary of the first sum $N - n - 1 = 4 - 3 - 1 = 0$ and it is the same as bottom one, so that first sum can be left out:

$$R_3 = \overline{A_0 A_3} = \sum_{i=0}^{1}\sum_{j=0}^{1} a_{0i} a_{3j}\cdot p_{0ij} =$$

$$= a_{00}\cdot a_{30}\cdot p_{000} + a_{00}\cdot a_{31}\cdot p_{001} + a_{01}\cdot a_{30}\cdot p_{010} + a_{01}a_{31}\cdot p_{011} =$$

$$= 0\cdot 0\cdot\frac{1}{4} + 0\cdot U\cdot\frac{1}{4} + U\cdot 0\cdot\frac{1}{4} + U\cdot U\cdot\frac{1}{4} = \frac{U^2}{4}. \qquad (23)$$

The joint probability p_{0ij}, $i, j = 0, 1$ denotes the probability of the simultaneous occurrence of amplitudes a_{0i} in the 1st pulse and a_{3j} in the 4th pulse of the period T. The random variables A_k ($k = 0, 3$) only acquire 2 values $a_{k0} = 0$ with the probability $p_{k0} = 1/2$ and $a_{k1} = U$ with the probability $p_{k1} = 1/2$ for $k = 0, 3$ within the period T. Then according to equation (7), it is:

$$p_{0ij} = p_{0i}\cdot p_{3j} = \frac{1}{2}\cdot\frac{1}{2} = \frac{1}{4}. \qquad (24)$$

Now the power spectral density of the alternate component can already be expressed using equation (2):

$$\mathbf{S}_3(f) = \frac{1}{4T_o}\left| T_o \frac{\sin\frac{\pi f}{f_o}}{\frac{\pi f}{f_o}} \right|^2 \sum_{n=-3}^{3} R_n \mathrm{e}^{-\mathrm{j}2\pi n f T_o} =$$

$$= \frac{T_o}{4}\cdot\left(\frac{\sin\frac{\pi f}{f_o}}{\frac{\pi f}{f_o}} \right)^2 \left(\frac{U^2}{4}\mathrm{e}^{\mathrm{j}6\pi f T_o} + \frac{U^2}{2} + \frac{U^2}{4}\mathrm{e}^{-\mathrm{j}6\pi f T_o} \right) =$$

$$= \frac{T_o}{4}\left(\frac{\sin\frac{\pi f}{f_o}}{\frac{\pi f}{f_o}}\right)^2 \frac{U^2}{2}\cdot\left(1+\frac{e^{j6\pi f T_o}+e^{-j6\pi f T_o}}{2}\right)=$$

$$= \frac{U^2}{4}T_o\left(\frac{\sin\frac{\pi f}{f_o}}{\frac{\pi f}{f_o}}\right)^2 \frac{1+\cos\frac{6\pi f}{f_o}}{2}=$$

$$= \frac{U^2}{4}T_o\left(\frac{\sin\frac{\pi f}{f_o}}{\frac{\pi f}{f_o}}\cdot\cos\frac{3\pi f}{f_o}\right)^2 . \qquad (25)$$

4) Spectrum of the Alternate Component of the Whole Signal

When the examined signal is composed of 3 independent signals a), b), c), the power spectral density of the whole signal is composed from the sum of power spectral densities of the particular signals. The double side power spectral density of the alternate component of the signal will then be:

$$S_a(f) = S_1(f) + S_2(f) + S_3(f) =$$

$$= 2\cdot\left(\frac{U}{4}\right)^2 T_o\left(\frac{\sin\frac{\pi f}{f_o}}{\frac{\pi f}{f_o}}\right)^2 + \frac{U^2}{4}T_o\left(\frac{\sin\frac{\pi f}{f_o}}{\frac{2\pi f}{f_o}}\cos\frac{3\pi f}{f_o}\right)^2 =$$

$$= \frac{U^2}{4}T_o\left(\frac{\sin\frac{\pi f}{f_o}}{\frac{\pi f}{f_o}}\right)^2\left(\frac{1}{2}+\cos^2\frac{3\pi f}{f_o}\right). \qquad (26)$$

5) Constant Component of the Signal

As $T = 4\cdot T_o$ ($N = 4$), the spectral component c_0 is given by the sum of the particular components of the signal a), b) and c):

$$c_0 = \frac{1}{4}+\frac{1}{4}+2\cdot\frac{1}{4} = 1 , \qquad (27)$$

and the spectrum of the constant component of the signal will be:

$$S_c(f) = m_a^2\cdot c_0^2 = \frac{U^2}{4}\cdot\delta(0) . \qquad (28)$$

6) The Entire Spectrum

This is given by the sum of the power spectral density of the alternate component and the spectrum of the constant component of the signal. Then the real spectrum will be:

$$S(f) = 2\cdot S_a(f) + S_c(f) =$$

$$= \frac{U^2}{2}T_o\left(\frac{\sin\frac{\pi f}{f_o}}{\frac{\pi f}{f_o}}\right)^2\left(\frac{1}{2}+\cos^2\frac{3\pi f}{f_o}\right)+\frac{U^2}{4}\delta(0) . \qquad (29)$$

3. MLT 3 Code

The encoding of a digital signal by the MLT 3 code is used in Ethernet on metallic shielded or unshielded twisted pairs at 100 Mbps bit rates. Any of 3 levels $-U$, 0, $+U$ can be assigned to the logical states 0 or 1 according to the following rule: If the logical 1 occurs, then the transition from the actual level to the next level always becomes. If the logical 0 occurs, then no transition takes place. The encoding is closer explained in Fig. 6 and in Tab. 2.

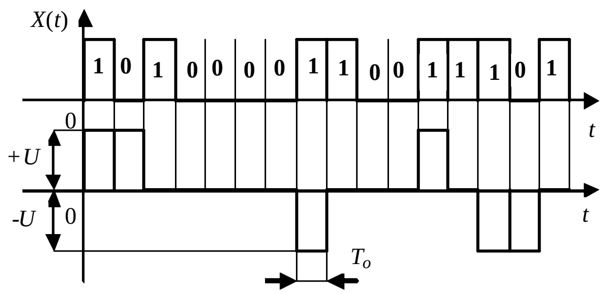

Fig. 6: MLT 3 Code.

Tab.2: States of the MLT 3 code.

Previous level	Next transition	New level
+U	$0 \to 0$	$+U$
	$0 \to 1$	0
	$1 \to 0$	$+U$
	$1 \to 1$	0
0	$0 \to 0$	0
	$0 \to 1$	$+U$ or $-U$
	$1 \to 0$	0
	$1 \to 1$	$+U$ or $-U$
-U	$0 \to 0$	$-U$
	$0 \to 1$	0
	$1 \to 0$	$-U$
	$1 \to 1$	0

The random amplitude A of the pulse acquires 3 discrete values in the MLT 3 code: $a_{-1} = -U$ with probability p_{-1} = 1/4, a_0 = 0 with probability p_0 = 1/2 and a_1 = +U with probability p_1 = 1/4 where probability of the logical 0 is p_O = 1/2 and probability of the logical 1 is p_I = 1/2.

The mean value of the amplitude of the signal with the MLT 3 code is zero because:

$$m_a = \sum_{j=-1}^{1} a_j \cdot p_j = -U \cdot \frac{1}{4} + 0 \cdot \frac{1}{2} + U \cdot \frac{1}{4} = 0, \quad (30)$$

and the dispersion of amplitudes at the zero mean value is:

$$\sigma_a^2 = \sum_{j=-1}^{1} a_j^2 p_j = (-U)^2 \frac{1}{4} + 0^2 \frac{1}{2} + U^2 \frac{1}{4} = \frac{U^2}{2}. \quad (31)$$

It can be seen from Tab. 2 that there is also a correlation coupling between adjacent pulses. The correlation coefficient R_0 has the same value as in (31). The correlation coefficient R_1 shall be calculated. As there is only the correlation between adjacent pulses, the correlation range N = 2, the upper boundary of the first sum in equation (6) $N - n - 1 = 2 - 1 - 1 = 0$, so that this sum can be omitted. Then like in (23), it is:

$$R_1 = A_0 \cdot A_1 = \sum_{i=-1}^{1} \sum_{j=-1}^{1} a_{0i} \cdot a_{1j} \cdot p_{oij} =$$

$$\begin{aligned} &= a_{0,-1} \cdot a_{1,-1} \cdot p_{0,-1,-1} + a_{0,-1} a_{1,0} \cdot p_{0,-1,0} + \\ &a_{0,-1} \cdot a_{1,1} \cdot p_{0-1,1} + a_{0,0} \cdot a_{1,-1} \cdot p_{0,0,-1} + \\ &a_{0,0} \cdot a_{1,0} \cdot p_{0,0,0} + a_{0,0} \cdot a_{1,1} \cdot p_{0,0,1} + \\ &a_{0,1} \cdot a_{1,-1} \cdot p_{0,1,-1} + a_{0,1} a_{1,0} \cdot p_{0,1,0} + \\ &a_{0,1} \cdot a_{1,1} \cdot p_{0,1,1} \end{aligned} \quad . \quad (32)$$

Now the probabilities $p_{0,i,j}$, i, j = –1, 0, 1 shall be calculated. The transitions from +U to –U and from –U to +U are not allowed according to Tab. 2. Then the probabilities $p_{0,1,-1}$ and $p_{0,-1,1}$ are zero:

$$p_{0,1,-1} = p_{0,-1,1} = 0. \quad (33)$$

An actual level –U, 0 or +U stays the same, if the bit change from 1 to 0 occurs or the bit 0 is not changed, the probabilities of which are:

$$p_{0,-1,-1} = p_{0,1,1} = p_{-1} \cdot p_I \cdot p_O + p_{-1} \cdot p_O \cdot p_O =$$

$$= p_1 \cdot p_I \cdot p_O + p_1 \cdot p_O \cdot p_O = \frac{1}{4} \cdot \frac{1}{2} \cdot \frac{1}{2} + \frac{1}{4} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{8}, \quad (34)$$

$$p_{0,0,0} = p_0 \cdot p_I \cdot p_O + p_0 \cdot p_O \cdot p_O =$$

$$= \frac{1}{4} \cdot \frac{1}{2} \cdot \frac{1}{2} + \frac{1}{4} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{8}. \quad (35)$$

An actual level –U, 0 or +U will be changed, if the bit change from 0 to 1 occurs or the bit 1 stays the same, the probabilities of which are:

$$p_{0,-1,0} = p_{0,1,0} = p_{-1} \cdot p_O \cdot p_I + p_{-1} \cdot p_I \cdot p_I =$$

$$= p_1 \cdot p_O \cdot p_I + p_1 \cdot p_I \cdot p_I = \frac{1}{4} \cdot \frac{1}{2} \cdot \frac{1}{2} + \frac{1}{4} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{8}, \quad (36)$$

$$\begin{aligned} &p_{0,0,-1} = p_{0,0,1} = p_0 \cdot p_O \cdot p_I + p_0 \cdot p_I \cdot p_I = \\ &= \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} + \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4} \end{aligned} \quad . \quad (37)$$

Thus the correlation coefficient R_1 will be:

$$\begin{aligned} R_1 &= (-U) \cdot (-U) \cdot \frac{1}{8} + (-U) \cdot 0 \cdot \frac{1}{8} + (-U) \cdot U \cdot 0 + \\ &+ 0 \cdot (-U) \cdot \frac{1}{4} + 0 \cdot 0 \cdot \frac{1}{4} + 0 \cdot U \cdot \frac{1}{4} + \\ &+ U \cdot (-U) \cdot 0 + U \cdot 0 \cdot \frac{1}{8} + U \cdot U \cdot \frac{1}{8} = \frac{U^2}{4}. \end{aligned} \quad (38)$$

As the signal does not contain either non-random or constant component, putting (38) to (2) we obtain for the entire spectrum of the code:

$$\mathbf{S}(f) = \frac{1}{T_o} \left| T_o \frac{\sin \frac{\pi f}{f_o}}{\frac{\pi f}{f_o}} \right|^2 \sum_{n=-1}^{1} R_n \cdot \mathrm{e}^{-\mathrm{j}2\pi f n T_o} =$$

$$= T_o \cdot \left(\frac{\sin \frac{\pi f}{f_o}}{\frac{\pi f}{f_o}} \right)^2 \left(\frac{U^2}{4} \cdot \mathrm{e}^{\mathrm{j}2\pi f T_o} + \frac{U^2}{2} + \frac{U^2}{4} \cdot \mathrm{e}^{-\mathrm{j}2\pi f T_o} \right) =$$

$$= T_o \left(\frac{\sin \frac{\pi f}{f_o}}{\frac{\pi f}{f_o}} \right)^2 \frac{U^2}{2} \left(1 + \cos \frac{2\pi f}{f_o} \right) =$$

$$= U^2 T_o \left(\frac{\sin \frac{\pi f}{f_o} \cdot \cos \frac{\pi f}{f_o}}{\frac{\pi f}{f_o}} \right)^2 = U^2 T_o \left(\frac{\sin \frac{2\pi f}{f_o}}{\frac{2\pi f}{f_o}} \right)^2. \quad (39)$$

The real spectrum will be:

$$S(f) = 2 \cdot (f) = 2 \cdot U^2 \cdot T_o \left(\frac{\sin \frac{2\pi f}{f_o}}{\frac{2\pi f}{f_o}} \right)^2. \quad (40)$$

4. AMI-NRZ Code

This code is used mainly on the S_0 bus of the ISDN basic access and slightly modified as the HDB 3 code is used in time division multiplex transmissions. Like the MLT 3 code, it is the code with 3 states where the random amplitude A of the pulse also acquires 3 discrete values: $a_{-1} = -U$ with probability $p_{-1} = 1/4$, $a_0 = 0$ with probability $p_0 = 1/2$ and $a_1 = +U$ with probability $p_1 = 1/4$ where probability of the logical 0 is $p_O = 1/2$ and probability of the logical 1 is $p_I = 1/2$. But there is the correlation coupling between the amplitudes $+U$ and $-U$, namely, when an amplitude U with the positive value has occurred in the past, then the next amplitude U which will occur after that will have the negative value and conversely, when an amplitude U with the negative value has occurred in the past, then the next amplitude U which will occur after that will have the positive value (Fig. 7, Tab. 3).

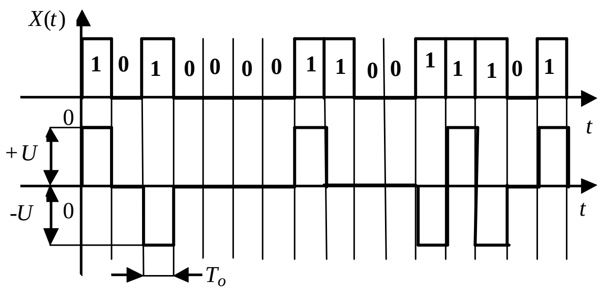

Fig. 7: AMI-NRZ code.

Similarly as at the MLT 3 code, the mean value m_a is also zero at the AMI code and the dispersion of amplitudes σ_a^2 and the covariance K_0 are given by (31).

Tab.3: States of the AMI code.

Previous level	Next transition	New level
+U	1	-U
	0	0
0	1	+U or –U
	0	0
-U	1	+U
	0	0

Let´s calculate the probabilities $p_{0,i,j}$, $i, j = -1, 0, 1$. It is not possible for the states $+U$ or $-U$ to occur next to each other according to Tab. 3. Then the probabilities $p_{0,-1,-1}$ and $p_{0,1,1}$ are zeros:

$$p_{0,-1,-1} = p_{0,1,1} = 0\,. \tag{41}$$

Futher:

$$p_{0,-1,0} = p_{0,0,-1} = p_{0,0,1} = p_{0,1,0} = \frac{1}{4}\cdot\frac{1}{2} = \frac{1}{8}. \tag{42}$$

$$p_{0,0,0} = \frac{1}{2}\cdot\frac{1}{2} = \frac{1}{4}. \tag{43}$$

The transition from the level $-U$ to the level $+U$ or conversely from $+U$ to $-U$ becomes when the bit with the logical 1 occurs. Then

$$p_{0,-1,1} = p_{0,1,-1} = p_{-1}\cdot p_{I/-1} = p_1\cdot p_{I/1} = \frac{1}{4}\cdot\frac{1}{2} = \frac{1}{8}. \tag{44}$$

So the correlation coefficient R_1 according to (32) will be:

$$\begin{aligned} R_1 &= (-U)\cdot(-U)\cdot 0 + (-U)\cdot 0\cdot\frac{1}{8} + (-U)\cdot U\cdot\frac{1}{8} + \\ &+ 0\cdot(-U)\cdot\frac{1}{8} + 0\cdot 0\cdot\frac{1}{4} + 0\cdot U\cdot\frac{1}{8} + \\ &+ U\cdot(-U)\cdot\frac{1}{8} + U\cdot 0\cdot\frac{1}{8} + U\cdot U\cdot 0 = -\frac{U^2}{4}. \end{aligned} \tag{45}$$

As the signal does not contain any either non-random or constant component, putting (45) into (2) we obtain for the entire spectrum of the code:

$$\begin{aligned} \boldsymbol{S}(f) &= \frac{1}{T_o}\left|T_o\frac{\sin\frac{\pi f}{f_o}}{\frac{\pi f}{f_o}}\right|^2 \sum_{n=-1}^{1} R_n\cdot \mathrm{e}^{-\mathrm{j}2\pi f n T_o} = \\ &= T_o\left(\frac{\sin\frac{\pi f}{f_o}}{\frac{\pi f}{f_o}}\right)^2\left(= -\frac{U^2}{4}\cdot\mathrm{e}^{\mathrm{j}2\pi f T_o} + \frac{U^2}{2} - \frac{U^2}{4}\cdot\mathrm{e}^{-\mathrm{j}2\pi f T_o}\right) = \\ &= T_o\left(\frac{\sin\frac{\pi f}{f_o}}{\frac{\pi f}{f_o}}\right)^2\frac{U^2}{2}\left(1-\cos\frac{2\pi f}{f_o}\right) = \\ &= U^2 T_o\left(\frac{\sin\frac{\pi f}{f_o}}{\frac{\pi f}{f_o}}\right)^2 \sin^2\frac{\pi f}{f_o}. \end{aligned} \tag{46}$$

The real spectrum will be:

$$S(f) = 2\cdot\mathbf{S}(f) = 2\cdot U^2\cdot T_o\left(\frac{\sin\frac{\pi f}{f_o}}{\frac{\pi f}{f_o}}\right)^2\sin^2\frac{\pi f}{f_o}. \tag{47}$$

5. Evaluation of Results

On Fig. 8, the spectra of the convolution encoded signal (29) (dotted line), the MLT 3 code (dashed line) and the AMI-NRZ code (dotted and dashed line) are compared with the basic random digital signal with the same pulse shape (full line) which can be according to (11) and (20) expressed as:

$$S_r(f)=\frac{U^2}{2}\cdot T_o\left(\frac{\sin\frac{\pi f}{f_o}}{\frac{\pi f}{f_o}}\right)^2+\frac{U^2}{4}\delta(0). \qquad (48)$$

The whole power of a signal can be calculated by 2 ways – either in the time domain:

$$P=\frac{1}{T}\int_{-\frac{\tau}{2}}^{\frac{\tau}{2}}\overline{X^2(t)}\mathrm{d}t=\frac{\overline{A^2}}{T_o}\int_{-\frac{\tau}{2}}^{\frac{\tau}{2}}f^2(t)\mathrm{d}t, \qquad (49)$$

or in the frequency domain by the integration of the power spectral density *S*(*f*) and/or applying Parseval theorem.

$$P=\int_0^\infty S(f)\mathrm{d}f+m_a^2\left(c_0^2+\frac{1}{2}\sum_{v=1}^\infty c_v^2\right). \qquad (50)$$

Applying these equations the powers of all examined signals can be calculated:

$$P=\frac{U^2}{2}. \qquad (51)$$

That means the entire power of the examined signals is not changed by coding.

One half of energy of the basic digital random signal and of the convolution encoded signal is concentrated in constant component c_0 which is:

$$P_0=\frac{U^2}{4}, \qquad (49)$$

as it can be deducted from equations (29), (48) and (50). The other half of energy is scattered over the alternated spectrum.

Random baseband digital signals are characterised by continuous power spectrum density curve. Due to the periodic deterministic component (rectangular pulses), the spectrum manifests a typical main lobe and diminishing side lobes. There are plotted only main lobes of the examined digital signals on Fig. 8. The main lobe on the convolution signal is deformed by coding. The power spectral densities curves gain zero values at spectral frequencies that equal to repeating frequency f_o and its multiples.

It can be interesting from the transmission point of view how much energy is in the main lobe which performs the bandwidth of the particular signal. It can be shown by the numerical computation of the integral in (50) in the range from 0 to f_o that 95 % of the whole power is scattered in the main spectrum lobe in case of the basic random digital signal, the convolution encoded signal and the MLT 3 code, and 86 % in case of the AMI-NRZ code. As to transmitted energy in trunked spectrum concerns, the AMI codes are less suitable for transmission than the other codes examined here.

6. Conclusion

Knowledge of spectra of digital signals carrying information is important both in radio communications at the frequency spectrum management, and in telecommunications to determine spectral compatibility of various transmission systems conveying information through physical circuits in metallic networks so that they are influenced each other by crosstalks as less as possible.

The spectrum calculation of a random digital signal applying (10) or (11) is quite simple. The goal of this contribution was to show the spectrum calculation in more complicated cases, when there is a correlation coupling among pulses of a signal.

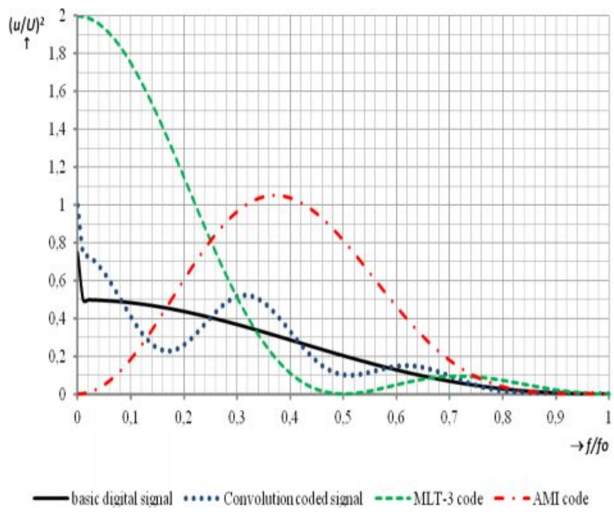

Fig. 8: Spectra comparison.

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